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Fuzzy Completely Weakly e -irresolute Functions

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Abstract—In this paper, we introduce a new class of functions called fuzzy completely e -irresolute functions between fuzzy topological spaces and also in this paper, fuzzy e -open sets and fuzzy e -closed sets are used to define and investigate a new class of functions called fuzzy completely weakly e -irresolute. Relationships between the new class and other classes of functionsare established.

Key words and phrases: Fuzzy topology, fuzzy e -open sets, fuzzy e –irresolute functions, fuzzy e - open set, fuzzy completely e - irresolute, fuzzy completely weakly e - irresolute, Fuzzy e - continuous, fuzzy e -connected.

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1 Introduction

EVER since the introduction of fuzzy sets by Zadeh [20], the fuzzy concepts has invaded almost all branches of Mathematics. The concept of fuzzy topological space has introduced by chang [5] in 1968. Since then many fuzzy topologists have extended various notions in classical topology to fuzzy topological spaces. In this paper, fuzzy e -open sets and fuzzy e -closed sets are used to define and investigate a new class of functions called fuzzy completely weakly e - irresolute. Relationships between the new class and other classes of functions are established. Throughout this paper X and Y are always fuzzy topological spaces. The class of all fuzzy sets on a universe X will be denoted by I^X. Let A be a fuzzy subset of a space X . The fuzzy closure of A, fuzzy interior of A, fuzzy δ -closure of A and the fuzzy δ -interior of A are denoted by $Cl(A)$, $Int(A)$, $Cl_\delta(A)$ and $Int_\delta(A)$ respectively.

A fuzzy subset A of space X is called fuzzy regular open [1] (resp. fuzzy regular closed) if $A = Int(Cl(A))$ (resp. $A = Cl(Int(A))$. The fuzzy δ -interior of fuzzy subset A of X is the union of all fuzzy regular open sets contained in A. A fuzzy subset A is called fuzzy δ -open [12] if $A = Int_\delta(A))$. The complement of fuzzy δ -open set is called fuzzy δ - closed (i.e,. $A = Cl_\delta(A))$

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2 Preliminaries

Now, we introduce some basic notions and results that are used in the sequel.

Definition 2.1. A fuzzy topology on a nonempty set X is a family δ of fuzzy subsets of X which satisfies the following three conditions:
(i) $0, 1 \in \delta$,
(ii) If g, $h \in \delta$, their $g \wedge h \in \delta$
(iii) $f_i \in \delta$ for each $i \in I$, then $\bigvee_{i\in I} f_i \in \delta$.
The pair (X, τ) is called a fuzzy topological space [5].

Definition 2.2. Members of δ are called fuzzy open sets [5] and complements of fuzzy open sets are called fuzzy closed sets [5], where the complement of a fuzzy set A, denoted by A^C, is $1 - A$.

Definition 2.3. [15] The fuzzy subset x_α of a non-empty set X , which $x \in X$ and $0 < a \leq 1$ defined by

$$x_a(p) = \begin{cases} a \ if \ \ p = x \\ 0 \ if \ \ p \neq x \end{cases}$$

is called a fuzzy point in X with suppost x and value a. The fuzzy point x_a is called point.

Definition 2.4. [15] Let λ be fuzzy set in X and x_a a fuzzy point in X. we say that $X_a \leq \lambda$.

Definition 2.5. [9] A fuzzy set λ of a fuzzy topological space X is said to be fuzzy γ-open if $\lambda \le Cl$ $(Int\ \lambda) \vee Int(Cl\ \lambda))$ where $Cl(\lambda) = \wedge\{\mu : \mu \ge \lambda, \mu$ is fuzzy closed in $X\}$ and $Int(\lambda) = \wedge\{\mu : \mu \le \lambda, \mu$ is fuzzy open in $X\}$. If λ is fuzzy γ-open, then $1-\lambda$ is fuzzy γ-closed.

Definition 2.6. [3] Let $f : X \to Y$ be a mapping. Then f is called a fuzzy γ-irresolute mapping if $f^{-1}(V)$ is a fuzzy γ-open set in X for each fuzzy γ-open set in Y.

Definition 2.7. [17] A fuzzy set λ of a fuzzy topological space X is said to be fuzzy e-open (resp. regular open [1]) if $\lambda \le Cl(Int_\delta \lambda) \vee Int(Cl_\delta \lambda))$ (resp. $\lambda = Int(Cl(\lambda))$) where $Cl(\lambda) = \wedge\{\mu : \mu \ge \lambda, \mu$ is fuzzy closed in $X\}$ and $Int(\lambda) = \wedge\{\mu : \mu \le \lambda, \mu$ is fuzzy open in $X\}$. If λ is fuzzy e-open, then $1-\lambda$ is fuzzy e-closed.

Definition 2.8. [17] Let X be a fuzzy topological space and λ be any fuzzy set in X. The fuzzy e-closure of λ in X is denoted by $eCl(\lambda)$ as follows:
$eCl(\mu) = \wedge\{\lambda : \lambda \ge \mu, \lambda$ is a fuzzy e -closed set of $X\}$. Similarly we can define $eInt(\lambda)$.

Remark 2.9. For a fuzzy set λ of X, $1-eInt(\lambda) = e\,Cl\ (1-\lambda)$.

Remark 2.10. A fuzzy set λ is fuzzy e-closed if and only if $eCl(\lambda) = \lambda$.

Definition 2.11. [15] A fuzzy set A in X is said to be q-coincident with a fuzzy set B, denoted by AqB, if there exists $x \in X$ such that $A(x) + B(x) > 1$. It is known that $A \le B$ if and only if A and $1-B$ are not q -coincident, denote by $A\ \overline{q}\ (1-B)$.

Definition 2.12. [15] A fuzzy set B is a quasi neighbourhood (q-neighbourhood, for short) of A if and only if there exists a fuzzy open set U such that $AqU \le B$.

Definition 2.13. A fuzzy set A in X is said to be a e-q-neighbourhood (e-q-nbd, for short) of x_α if and only if there a fuzzy e -open set V in X such that $x_\alpha qV \le A$.

Theorem 2.14. [15] In a fuzzy topological space X, λ be a fuzzy e-closed (resp. fuzzy e-open) if and only if $\lambda = eCl(\lambda)$ (resp. $\lambda = eInt(\lambda)$).

Definition 2.15. [3] Let X and Y be two fuzzy topological spaces. Let $\lambda \in I^X, \mu \in I^Y$. Then $f(\lambda)$ is a fuzzy subset of Y, defined by $f(\lambda) : Y \to [0,1]$

$$f(\lambda)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(\{y\})} \lambda(x) & if\ \ f^{-1}(\{y\}) \ne \phi \\ 0 & if\ \ f^{-1}(\{y\}) = \phi \end{cases}$$

and $f^{-1}(\mu)$ is a fuzzy subset of X, defined by $f^{-1}(\mu)(x) = \mu(f(x))$.

Lemma 2.16. [1] Let $f : X \to Y$ be a function and $\{\lambda_\alpha\}$ be a family of fuzzy sets of Y, then
(i) $f^{-1}(\bigcup \lambda_\alpha) = \bigcup f^{-1}(\lambda_\alpha)$,
(ii) $f^{-1}(\bigcap \lambda_\alpha) = \bigcap f^{-1}(\lambda_\alpha)$.

Lemma 2.17. [1] For functions $f_i : X_i \to Y_i$, and fuzzy sets λ_i of Y_i, i = 1, 2; we have $(f_1 \times f_2)^{-1}(\lambda_1 \times \lambda_2) = f_1^{-1}(\lambda_1) \times f_2^{-1}(\lambda_2)$.

Lemma 2.18. [1] Let $g : X \to X \times Y$ be the graph of a function $f : X \to Y$. Then, if λ is a fuzzy set of X and μ is a fuzzy set of Y. $g^{-1}(\lambda \times \mu) = \lambda \wedge f^{-1}(\mu)$.

Definition 2.19. A functions $f : X \to Y$ is said to be:
1. fuzzy completely continuous [4] if $f^{-1}(V)$ is fuzzy regular open in X for each fuzzy open set V in Y;
2. fuzzy e-irresolute [16] if $f^{-1}(V)$ is fuzzy e-open in X for each fuzzy e–open set V in Y;
3. fuzzy e-continuous [17] if $f^{-1}(V)$ is fuzzy e-open in X for each fuzzy open set V in Y;
4. fuzzy totally continuous [11] if $f^{-1}(V)$ is fuzzy clopen in X for each fuzzy subset V in Y;
5. fuzzy open [19] if $f(V)$ is fuzzy open set in Y for each fuzzy open set V in X;
6. fuzzy almost open [13] if $f(V)$ is fuzzy regular open set in Y for each fuzzy regular open set V in X;
7. fuzzy strongly continuous [2] if $f^{-1}(V)$ is fuzzy open fuzzy

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closed set in X for every fuzzy set λ in Y.

Definition 2.20. A function $f: X \to Y$ is called fuzzy e-open [16] (resp. fuzzy pre- e -open) if the image of each fuzzy open (resp. fuzzy e -open) set in X is fuzzy e -open in Y.

Definition 2.21. [2] A function $f: X \to Y$ is called fuzzy completely continuous if $f^{-1}(V)$ is fuzzy regular open in X for every fuzzy open set V of Y.

Definition 2.22. [16] A function $f: X \to Y$ is called fuzzy e-irresolute (resp. Fuzzy e-continuous) $f^{-1}(V)$ is fuzzy e-open in X for every fuzzy e-open (resp. fuzzy open) set V of Y.

Definition 2.23. A space (X,τ) is called fuzzy nearly compact [10] (resp.fuzzy e-compact) if every fuzzy regular open (resp. fuzzy e-open) cover of X has a finite subcover.

Definition 2.24. [18] A space X is called fuzzy almost normal if for each fuzzy closed set A and each fuzzy regular closed set B such that $A \cap B = \phi$, there exists disjoint fuzzy open sets U and V such that $A \le U$ and $B \le V$.

3 Fuzzy Completely e - Irresolute Function

Definition 3.1. Let (X,τ) and (Y,σ) be a fuzzy topological spaces. A function $f:(X,\tau) \to (Y,\sigma)$, is said to be a fuzzy completely e-irresolute function if $f^{-1}(V)$ is fuzzy regular open in X for every fuzzy e-open set λ of Y .

Remark 3.2. Every fuzzy strongly continuous function is fuzzy e -irresolute, but the converse is not true.

Example 3.3 Let $X = Y = \{a,b,c\}$. Define fuzzy sets $\mu_1, \mu_2 : X \to [0,1]$ such that $\tau = \{0,1\}$ and $\sigma = \{0,1,\mu_1,\mu_2\}$ where $\mu_1 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.5}{c}$, $\mu_2 = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.5}{c}$ Define $f:(X,\tau) \to (Y,\sigma)$ be the identity function. Then f is fuzzy e –irresolute but not fuzzy strongly continuous.

Remark 3.4. Every completely e -irresolute function is fuzzy e-irresolute. But the converse is not true.

Example 3.5 Let $X = Y = \{a,b,c\}$. Define fuzzy sets $\mu_1, \mu_2 : X \to [0,1]$ such that $\tau = \{0,1,\mu_3\}$ and $\sigma = \{0,1,\mu_1,\mu_2\}$ where $\mu_1 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.5}{c}$, $\mu_2 = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.5}{c}$. Define $f:(X,\tau) \to (Y,\sigma)$ be the identity function. Then f is fuzzy e -irresolute but not fuzzy completely e -irresolute.

Remark 3.6. Every e-irresolute function is fuzzy e-irresolute. But the converse is not true.

Example 3.7. Let $X = Y = \{a,b,c\}$. Define fuzzy sets $\mu_1, \mu_2, \mu_3, \mu_4 : X \to [0,1]$ such that $\tau = \{0,1, \mu_1, \mu_2, \mu_3, \mu_4\}$ and $\sigma = \{0,1,\mu_5\}$ where $\mu_1 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.5}{c}$ $\mu_2 = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.5}{c}$, $\mu_3 = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.4}{c}$, $\mu_4 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.4}{c}$, $\mu_5 = \frac{0.4}{a} + \frac{0.5}{b} + \frac{0.6}{c}$. Define $f:(X,\Im) \to (Y,\sigma)$ be the identity function. Then $\lambda = \frac{0.7}{a} + \frac{0.6}{b} + \frac{0.4}{c}$ is fuzzy open but not e-open in (X,τ). Therefore f is fuzzy γ-irresolute but not fuzzy completely e -irresolute.

Theorem 3.8. If $f:(X,\tau) \to (Y,\sigma)$ is a fuzzy completely e-irresolute function A is any fuzzy open subset of X, then the restriction $f|_A : A \to Y$ is fuzzy completely e-irresolute.

Proof. Let λ be a fuzzy e-open subset of Y. By hypothesis, $f^{-1}(\lambda)$ is fuzzy regular open in X . Since A is fuzzy open in X , Then $(f|_A)^{-1}(\lambda) : f^{-1}(\lambda) \cap A$ is fuzzy regular open in A. Therefore, $f|_A$ is fuzzy completely e-irresolute.

Theorem 3.9. The following hold for functions $f: X \to Y$ and $g: Y \to Z$:

1. If $f: X \to Y$ is fuzzy completely e-irresolute and $g: Y \to Z$ is fuzzy e-irresolute, then $g \circ f: X \to Z$ is fuzzy completely e-irresolute.
2. If function $f: X \to Y$ is fuzzy completely continuous and is fuzzy completely e-irresolute, then $g \circ f: X \to Z$ is

fuzzy completely e -irresolute.

3. If $f : X \to Y$ is fuzzy completely e-irresolute and $g : Y \to Z$ is fuzzy e-continuous, then $g \circ f : X \to Z$ is fuzzy completely continuous.

Proof. Obvious.

Definition 3.10. A space X is said to be fuzzy e-connected, if X cannot be expressed as the union of two nonempty fuzzy e-open sets.

Theorem 3.11. If a mapping $f : X \to Y$ is fuzzy completely e -irresolute surjection and X is fuzzy almost connected then Y is fuzzy e -connected.

Proof. Assume that X is fuzzy connected and Y is not fuzzy e -connected. Then Y can be written as $Y = U \cup V$ such that U and V are disjoint nonempty fuzzy e-open sets. Since f is fuzzy completely e-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$) are disjoint fuzzy regular open sets and $X = f^{-1}(U) \cup f^{-1}(V)$ This shows that X is not fuzzy connected. This is a contradiction.

Definition 3.12. A space X is called fuzzy almost regular [6] (resp. fuzzy strongly e-regular) if for any fuzzy regular closed (resp. fuzzy e-closed) set $F \le X$ and any point $x \in X - F$, there exists disjoint fuzzy open (resp. fuzzy e -open) sets U and V such that $x \in U$ and $F \le V$.

Definition 3.13. A function $f : X \to Y$ is called fuzzy pre- e -closed if the image of every fuzzy e-closed subset of X is fuzzy e-closed set in Y .

Theorem 3.14. If a mapping $f : X \to Y$ is fuzzy pre-e-closed, then for each subset B of Y and a fuzzy e-open set U of X containing $f^{-1}(B)$ there exists a fuzzy e-open set V in Y containing B such that $f^{-1}(V) \le U$.

Proof. Obvious.

Theorem 3.15. If f is fuzzy completely e-irresolute e-open from an almost regular space X onto a space Y , then Y is fuzzy strongly f -regular.

Proof. Let f be fuzzy e-closed set in Y with $y \notin F$ such that $y = f(x)$. Since f is fuzzy completely e-irresolute function, $f^{-1}(F)$ is fuzzy regular closed and so fuzzy closed set in X and hence $x \notin f^{-1}(F)$. By almost regularity of X there exists disjoint fuzzy open sets U and V such that $x \in U$ and $f^{-1}(F) \le V$. We obtain that $y = f(x) \in f(U)$ and $F \le f(V)$ such that $f(U)$ and $f(V)$ are disjoint fuzzy e-open sets. Thus Y is fuzzy strongly e-regular.

Definition 3.16. A space X is called fuzzy strongly e -normal if for every pair of disjoint fuzzy e-closed subsets F_1 and F_2 of X there exists disjoint fuzzy e –open sets U and V such that $F_1 \le U$ and $F_2 \le V$.

Theorem 3.17. If f is fuzzy completely e-irresolute injective function from an fuzzy almost normal spaces X onto a space Y then Y is fuzzy strongly e-normal.

Proof. Let F_1 and F_2 be disjoint fuzzy e-closed sets in Y. Since f is fuzzy completely e-irresolute function $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint fuzzy regular closed and so fuzzy closed set in X . By fuzzy almost normality of X , there exists disjoint fuzzy open sets U and V such that $f^{-1}(F)_1 \le U$ and $f^{-1`}(F_2) \le V$. We obtain that $F_1 \le U$ and $F_2 \le V$. such that $f(U)$ and $f(V)$ are disjoint fuzzy e-open. Thus Y is fuzzy strongly e-normal.

Definition 3.18. A fuzzy topological space (X, τ) is said to be fuzzy e-T_1 (resp. fuzzy r-T_1) if for each pair of distinct points x and y of X , there exists fuzzy e-open (resp. fuzzy regular open) sets U_1 and U_2 such that $x \in U_1$ and $y \in U_2$, $x \notin U_2$ and $y \notin U_1$.

Theorem 3.19. If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy completely e -irresolute injective function and Y is fuzzy e-T_1 then X is fuzzy r-T_1.

Proof. Suppose that Y is fuzzy e-T_1. For any two distinct points x and y of X , there exists fuzzy e-open sets F_1 and F_2 in Y such that $f(x) \in F_1$, $f(y) \in F_2$, $f(x) \notin F_2$ and $f(y) \notin F_1$. Since f injective fuzzy completely e -irresolute function, we have X is fuzzy r-T_1.

Definition 3.20. A fuzzy topological space (X, τ) is said to be fuzzy e-T_1 (resp. fuzzy r-T_1) if for each pair of distinct points x and y of X, there exists disjoint fuzzy e-open (resp. fuzzy regular open) sets A and B such that $x \in A$ and $y \in B$.

Theorem 3.21. If $f:(X,\tau)\to(Y,\sigma)$ is fuzzy completely e-irresolute injective function and Y is fuzzy e-T_2 then X is fuzzy r-T_2.

Proof. Suppose that Y is fuzzy e - T_2. For any two distinct points x and y of X , there exists fuzzy e-open sets F_1 and F_2 in Y such that $f(x)\in F_1$, $f(y)\in F_2$, $f(x)\notin F_2$ and $f(y)\notin F_1$. Since f injective fuzzy completely e -irresolute function, we have X is fuzzy r-T_1.

4 Fuzzy Completely weakly e -irresolute Function

Definition 4.1. A function $f:(X,\tau)\to(Y,\sigma)$ is fuzzy completely weakly e-irresolute if and only if the inverse image of each fuzzy e-open set V in Y is fuzzy open set in X.

It is evident that every fuzzy completely e-irresolute function is fuzzy completely weakly e-irresolute function and every completely weakly e-irresolute function is fuzzy e -irresolute.

However, none of the above implications are not true as shown in the following example.

Example 4.2 Let $I=[0,1]$ and μ_1 and μ_2 be fuzzy subsets of I defined as

$$\mu_1(x)=\begin{cases}\frac{1}{5}(6x+1)\ if\ 0\le x\le\frac{1}{4}\\ \frac{1}{3}(2x+1)\ if\ \frac{1}{4}\le x\le 1\end{cases}$$

$$\mu_2(x)=\begin{cases}\frac{1}{10}(4x+1)\ if\ 0\le x\le\frac{1}{4}\\ \frac{4}{3}(1-x)\ if\ \frac{1}{4}\le x\le 1\end{cases}$$

Clearly $\tau_1=\{0,1\}$ and $\tau_2=\{0,1,\mu_1\}$ and $\tau_3=\{0,1,\mu_1,\mu_2,$ $\mu_1\vee\mu_2$, $\mu_1\wedge\mu_2\}$ are topologies on I . Let $f:(I,\tau_1)$ $\to(I,\tau_2)$ be defined by $f(x)=x$ for each $x\in I$. Then f is fuzzy e-irresolute but not fuzzy completely weakly e-irresolute.

Let $g:(I,\tau_3)\to(I,\tau_2)$ be defined by $g(x)=x$ for each $x\in I$. Then $g^{-1}=(1)$, $g^{-1}(\mu_2)=(\mu_2)$ which is fuzzy open but not regular open in (I,τ_3) Therefore, g is fuzzy completely weakly e -irresolute but not fuzzy completely e-irresolute.

Theorem 4.3. For a function $f:(X,\tau)\to(Y,\sigma)$, the following statements are equivalent:

(i) f is fuzzy completely weakly e-irresolute;

(ii) for each fuzzy point x_α in X and each fuzzy e-open e-q-nbd V of $f(x_\alpha)$, there exists a fuzzy open q-nbd U of x_α subset that $f(U)\le V$;

(iii) $f(Cl(A))\le eCl(f(A))$, for each fuzzy set A in X ;

(iv) $Cl(f^{-1}(B))\le f^{-1}(eCl(B))$, , for each fuzzy set B in Y;

(v) for each fuzzy e -closed set V in Y, $f^{-1}(V)$ is fuzzy closed set in X;

(vi) $f^{-1}(e\text{-}Int(B))\le Int(f^{-1}(B))$, for each fuzzy set B in Y.

Proof. (i)⇒(ii). Let V be any fuzzy e-open e-q -nbd of $f(x_\alpha)$ in Y. Then $V(f(x))+\alpha>1$. We choose a positive real number δ such that $V(f(x))>\delta>1-\alpha$ Then V is a fuzzy e -open set, $f(x_\alpha)\in V$. By hypothesis, there exists fuzzy open set U, $x_\alpha\in U$ such that $f(U)\le V$, $U(X)>$ $\delta>1-\alpha$. Therefore, U is a fuzzy open q -nbd of x_α.

(ii) ⇒ (iii). Let $x_\alpha\in Cl(A)$ then UqA and $f(U)qf(A)$ implies $Vqf(A)$, $f(x_\alpha)\in eCl(f(A))$ and $x_\alpha f^{-1}(eCl$ $(f(A))$. Therefore, $Cl(A)\le f^{-1}(eCl(f(A)))$. Hence, $f(x_\alpha)\in eCl(f(A))ff^{-1}(eCl(f(A)))\le eCl(f(A))$.

(iii) ⇒ (iv). Clear

(iv) ⇒ (ii). Let x_α be a fuzzy point in X and V be a fuzzy e -open e-q-nbd of $f(x_\alpha)$ and let $f(x_\alpha)\notin eCl(1-V)$, otherwise since V is a fuzzy e-open e - q –nbd of $f(x_\alpha)$, we have $Vq(1-V)$ which is a contradiction. Thus, $x_\alpha\notin f^{-1}(eCl(1-V))$ and by hypothesis, $x_\alpha\ne Cl$ $(f^{-1}(1$ $-U))$. Then there exists a fuzzy open set U of x_α such that $U\bar{q}\ f^{-1}(1-V)$ which implies that $f(U)\le V$.

(iv)⇒(v). Let F be any e -closed set in Y. By (iv), we have $Cl(f^{-1}(F))\le f^{-1}(eCl(F))=f^{-1}(F)$ and so, $f^{-1}(F)$ is fuzzy closed in X .

(iv) ⇒ (v) Clear.

(i) ⇒ (vi) For any fuzzy set V in Y, e- $Int(V)$ is fuzy e-open set in Y and so $f^{-1}(eInt(V))$ is fuzzy open set in X .

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Hence $f^{-1}(eInt(V) = Int(f^{-1}(eInt(V))) \leq Int(f^{-1}(V))$.

(vi) ⇒ (i). Obvious.

Theorem 4.4. Let X and Y be fuzzy topological space such that X is product related to Y. Then the product $U \times V$ of a fuzzy e-open set U in X and fuzzy e-open set V in Y. is a fuzzy e-open set in the fuzzy product space.

Proof. Similar to the proof of Theorem 3.10 in [1]

Theorem 4.5. If $f_i : X_i \to Y_i$ (i = 1, 2) are fuzzy completely weakly e–irresolute functions and Y_1 is product related to Y_2, then $f_i : X_1 \times X_2 \to Y_1 \times Y_2$ is fuzzy completely weakly e-irresolute.

Proof. Consider $A = \vee(U_i \times V_i)$ where U_i's and V_i 's are fuzzy e-open sets of Y_1 and Y_2, respectively. Since Y_1 is producted to Y_2, then from Theorem 4.2., A is fuzzy e -open set of $Y_1 \times Y_2$. By Lemma 2.1. and 2.2., $f^{-1}(A) = \vee(f_1^{-1}(U_i) \times f_2^{-2}(V_i))$. Since f_1 and f_2 are completely weakly e-irresolute, $f^{-1}(A)$ is a fuzzy open in $X_1 \times X_2$.

Theorem 4.6. Let $f : (X, \tau) \to (Y, \sigma)$ be a function and X is product related to Y. If the graph $g : X \to X \times Y$ of f is fuzzy completely weakly e-irresolute, then so is f.

Proof. Let V be a fuzzy e-open set in Y. By Lemma 2.3., we have $f^{-1}(V) = 1 \wedge f^{-1}(V) = g^{-1}(1 \times V)$. Since g is fuzzy completely weakly e-irresolute and $1 \times V$ is fuzzy e-open set in $X \times Y$. $f^{-1}(V)$ is fuzzy open set in X and so, f is fuzzy completely weakly e -irresolute.

Next, the composition and preservation of fuzzy topological structure under the fuzzy completely weakly e-irresolute which other fuzzy functions are studied.

The proof of the following theorem is obvious and hence omitted.

Theorem 4.7. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two functions.

1. If is fuzzy completely weakly e-irresolute and is g fuzzy completely e-irresolute, then $g \circ f$ is fuzzy completely e-irresolute.
2. If f is fuzzy completely weakly e-irresolute and g is fuzzy e-irresolute, then $g \circ f$ is fuzzy completely weakly $g \circ f$ -irresolute.
3. If f is fuzzy completely continuous and g is fuzzy completely weakly e-irresolute, then $g \circ f$ is fuzzy completely e-irresolute.
4. If f is fuzzy completely e-irresolute and g is fuzzy completely weakly e-irresolute, then $g \circ f$ is fuzzy completely e-irresolute.
5. If f is fuzzy totally continuous and g is fuzzy completely weakly e-irresolute, then $g \circ f$ is fuzzy completely e-irresolute.
6. If f is fuzzy completely weakly e-irresolute and g is fuzzy e -continuous, then $g \circ f$ is fuzzy continuous.
7. If f is fuzzy e-continuous and g is fuzzy completely weakly e -irresolute, then $g \circ f$ is fuzzy e-irresolute.
8. If f is fuzzy continuous and g is fuzzy completely weakly e-irresolute, then $g \circ f$ is fuzzy completely weakly e-irresolute.

Proof. Obvious.

Theorem 4.8. Let $f : (X, \tau) \to (Y, \sigma)$ is fuzzy almost open surjective function and $g : (Y, \sigma) \to (Z, \eta)$ is function such that $g \circ f : (X, \tau) \to (Z, \eta)$ is fuzzy completely e-irresolute, then g is fuzzy completely weakly e-irresolute.

Proof. Let V be a fuzzy e-open set in Z . since $g \circ f$ is fuzzy completely e-irresolute, $(g \circ f)^{-1}(V) = f^{-1}g^{-1}(V))$ is fuzzy regular open in X. Since f is fuzzy almost open surjective, $f(f^{-1}(g^{-1}(V)) = g^{-1}(V)$ is fuzzy open in Y. Therefore, g is fuzzy completely weakly e -irresolute.

Theorem 4.9. Let $f : (X, \tau) \to (Y, \sigma)$ is fuzzy open surjective function and $g : (Y, \sigma) \to (Z, \eta)$ is a function such that $g \circ f : (X, \tau) \to (Z, \eta)$ is fuzzy completely weakly e-irresolute, then g is fuzzy completely weakly e-irresolute.

Proof. Similar to the proof of Theorem 4.6.

Theorem 4.10. Let P_i be the projection function from $\prod X_i$ onto X_i. If $f : X \to \prod X_i$ is fuzzy completely weakly e -irreolute function, then $P_i \circ f$ is also fuzzy completely weakly e-irresolute.

Proof. Obvious.

Definition 4.11. A collection μ of fuzzy sets in a fuzzy space X is said to be cover [8] of a fuzzy set η of X if and only if

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$\left(\bigvee_{A\in\mu} A\right)(x)=1,$ for every $x \in S(\eta)$. A fuzzy cover μ of a fuzzy set η in a fuzzy space X is said to have a finite subcover if and only if there exists a finite subcollection $\rho=\{A_1, A_2,...,A_n$ of μ such that $\left(\bigvee_{A\in\mu} A\right)(x) \ge \eta(x),$ for every $x \in s(\eta)$, where $s(\eta)$ denotes the support of a fuzzyset η.

Definition 4.12. A fuzzy topological space X is called:
1. fuzzy compact [7] if every fuzzy open cover λ of X has a finite subcover.
2. fuzzy e -compact [16] if every fuzzy e -open cover λ of X has a finite subcover.

3. fuzzy e-closed [16] if every fuzzy e -open cover λ of X has a finite subfamily V of λ such that $(\bigvee_{u\in V} eCl(u))(x)=1$ for each $x \in X$.

Theorem 4.13. Let $f:(X,\tau)\to(Y,\sigma)$ is a fuzzy completely weakly e-irresolute surjective function and X is fuzzy compact space, then Y is fuzzy e-compact.
Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be any fuzzy e-open cover of Y. Then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is a fuzzy open cover of X. Since X is fuzzy compact there exists a finite subfamily $f^{-1}(V_\alpha) : i = 1, 2,..., n\}$ of $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ which covers X. Hence, Y is fuzzy e-compact.

Corollary 4.14. Let $f:(X,\tau)\to(Y,\sigma)$ is fuzzy completely weakly e –irresolute surjective function and X is fuzzy compact space, then Y is fuzzy e-closed.

Definition 4.15. Two non-zero fuzzy sets A and B in X are said to be separated [13] (resp. fuzzy e-separated) if $A\bar{q}\,Cl(B)$ and $B\bar{q}\,Cl(A)$ (resp. $A\bar{q}\,eCl(B)$ and $B\bar{q}\,eCl(A)$).

Definition 4.16. A fuzzy topological space X is said to be fuzzy connected [14] (resp. fuzzy e-connected) if it cannot be expressed as the union of two fuzzy separated (resp. fuzzy e-separated) sets.

Lemma 4.17. Two non-zero fuzzy sets A and B are fuzzy e-separated if and only if there exist two fuzzy e-open sets U and V such that $A \le U$, $B \le V$, AqV and $Bq\,U$.

Theorem 4.18. Let $f:(X,\tau)\to(Y,\sigma)$ is fuzzy completely weakly e–irresolute surjective function. If U is a fuzzy connected subset in X, then $f(U)$ is fuzzy e-connected in Y.
Proof. Suppose that $f(U)$ is not e-connected in Y. Then there exist fuzzy e-separated subsets G and H in Y. such that $f(U)=G\vee H$. By Lemma 4.1., thereexist fuzzy e-open subsets V and W such that $G\le V$, $H \le W$, $G\bar{q}\,W$ and $H\bar{q}V$. Since f is fuzzy complete weakly e -irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are fuzzy open in X and $U=f^{-1}(f(U))=f^{-1}(G\vee H)=f^{-1}(G)\vee f^{-1}(H)$. It is clear that $f^{-1}(G)$ and $f^{-1}(H)$ are fuzzy separated in X. Therefore, U is not fuzzy connected in X.

Corollary 4.19. Let $f:(X,\tau)\to(Y,\sigma)$ be a fuzzy completely weakly e –irresolute surjective function. If U is a fuzzy connected subset in X, then $f(U)$ is also fuzzy e-connected.

Theorem 4.20. A function $f : X \to Y$ is fuzzy completely weakly e -irresolute if the graph function $g : X \to X\times X$, defined by $g(x)=(x, g(x))$ for each $x\in X$ is fuzzy com pletely weakly e -irresolute .
Proof. Let V be any fuzzy e-open set of Y . Then $1\times V$ is a fuzzy e -open set of $X\times Y$. Since g is fuzzy completely e -irresolute, $f^{-1}(V)=g^{-1}(1\times V)$ is fuzzy regular open in X . Thus f is fuzzy completely weakly e-irresolute.

Theorem 4.21. If $f:(X,\tau)\to(Y,\sigma)$ is fuzzy completely weakly e–irresolute injective function and Y is fuzzy e - T_1 then X is fuzzy Hausdorff.
Proof. Let x, y be any two distinct points of X. Since f is injective, we have $f(x)\ne f(y)$. Since Y is fuzzy e - T_2 , there exists V and W are e -open sets in Y. such that $V\wedge W=0$. Since f is fuzzy completely weakly e-irresolute, there exists fuzzy open sets G and H in X such that $f(G)\le V$ and $f(H)\le W$. Hence we obtain $G\wedge H=0$. This shows that X is fuzzy Hausdorff.

Theorem 4.22. If a function $f : X \to Y$ is a fuzzy completely weakly e –irresolute surjection and X is fuzzy connected, then Y is fuzzy e -connected.
Proof Suppose that Y is not fuzzy e-connected. There exists non empty fuzzy e -open sets V and W of Y such that $Y=V\vee W$. Since f is fuzzy completely weakly e -irresolute

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$f^{-1}(V)$ and $f^{-1}(W)$ are fuzzy open sets and $X = f^{-1}(V) \vee f^{-1}(W)$. This shows that X is not fuzzy connected. This is a contradiction.

Conclusion

We have defined and proved basic properties of Fuzzy Completely e-Irresolute Functions and Fuzzy Completely Weakly e-Irresolute Function. Many results have been established to show how far topological structures are preserved by these e-Irresolute Functions. We also have provided examples where such properties fail to be preserved.

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